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a. $(C^T B) A^T$

b. $(2D^T - E) A$

$$C^T = \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \quad C^T B = \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$(C^T B) A^T = \begin{bmatrix} 4 & 5 \\ 16 & -2 \\ 8 & 8 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$(C^T B) A^T = \begin{bmatrix} 12 & 6 & 5 \\ 48 & -20 & 17 \\ 24 & 8 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -4 & 2 & 1 \\ 2 & -1 & 7 & 5 \\ -1 & 1 & 2 & 7 \\ 0 & -2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -32 \\ 18 \\ 17 \\ -2 \end{bmatrix}$$

$$\begin{array}{l} f_{21} \\ f_{31} \\ f_{41} \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -1 & 0 & 2 & 1 \end{bmatrix} \begin{array}{l} R_2 = R_2 - f_{21}(R_1) \\ R_3 = R_3 - f_{31}(R_1) \\ R_4 = R_4 - f_{41}(R_1) \end{array} \begin{bmatrix} -1 & -4 & 2 & 1 \\ 0 & -5 & 11 & 11 \\ 5 & 1 & 0 & 0 \\ 0 & 7 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{l} f_{32} \\ f_{42} \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ -1 & -6 & 2 & 1 \end{bmatrix} \begin{array}{l} R_3 = R_3 - f_{32}(R_2) \\ R_4 = R_4 - f_{42}(R_2) \end{array} \begin{bmatrix} -1 & -4 & 2 & 1 \\ 0 & -5 & 11 & 11 \\ 0 & 7 & 11 & 6 \\ 0 & -4 & 32 & -7.33 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ -1 & 0.67 & 4.33 & 4 \end{bmatrix} \begin{array}{l} R_4 = R_4 - f_{43}(R_3) \\ f_{42} \end{array} \begin{bmatrix} -1 & -4 & 2 & 1 \\ 0 & -5 & 11 & 11 \\ 0 & 7 & 11 & 6 \\ 0 & -3 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -0.56 & 1 & 0 \\ 1 & 0.67 & 0.61 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -32 \\ 14 \\ 11 \\ -4 \end{bmatrix}$$

$$LUX = B$$

$$ZUX = Z$$

$$LZ = D$$

$$z_1 = -32$$

$$-2z_1 + z_2 = 14$$

$$z_1 - 0.56z_2 + z_3 = 11$$

$$z_1 + 0.67z_2 + 0.61z_3 + z_4 = -4$$

$$z_1 = -32$$

$$z_2 = \cancel{14} - 50$$

$$z_3 = \cancel{0} + 15.22$$

$$z_4 = \cancel{14} + 6.61$$

$$\begin{bmatrix} -1 & -4 & 2 & 1 \\ 0 & -5 & 11 & 11 \\ 0 & 0 & 7.11 & 6.11 \\ 0 & 0 & 0 & -3.61 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -32 \\ -50 \\ 15.22 \\ 6.61 \end{bmatrix}$$

$$-x_1 - 4x_2 + 2x_3 + x_4 = -32$$

$$-5x_2 + 11x_3 + 11x_4 = -50$$

$$7.11x_3 + 6.11x_4 = 15.22$$

$$-3.61x_4 = 6.61$$

$$x_1 = 6.17$$

$$x_2 = 7.86$$

$$x_3 = 3.71$$

$$x_4 = -1.83$$

$$x = \begin{vmatrix} 8 & 1 & 2 \\ 11 & 1 & 4 \\ 12 & 4 & 1 \end{vmatrix}$$

$$= \frac{-13}{-15} = 1$$

$$\begin{vmatrix} 2 & 1 & 2 \\ 1 & 1 & 4 \\ 2 & 4 & 1 \end{vmatrix}$$

$$y = \begin{vmatrix} 8 & 2 \\ 11 & 4 \\ 12 & 1 \end{vmatrix}$$

$$= \frac{-38}{-15} = 2$$

$$z = \begin{vmatrix} 2 & 1 & 8 \\ 1 & 1 & 11 \\ 2 & 4 & 12 \end{vmatrix}$$

$$= \frac{-38}{-15} = 2$$

$$\begin{array}{l} A = 1 \\ B = 2 \\ C = 2 \end{array}$$

4.

$$\begin{vmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{vmatrix}$$

Characteristic equation:

$$\begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{vmatrix} \begin{array}{l} |A - \lambda I| = 0 \\ \lambda \rightarrow \\ 3-5-\lambda \\ 6 \neq 0 \end{array}$$

$$6(-5-\lambda)(3) + (-6)(3)(1-\lambda) + (4-\lambda)(3)(3)$$

$$- \left[(1-\lambda)(-5-\lambda)(4-\lambda) - 3(3)(6) + 3(3)(-6) \right]$$

$$x^3 - 10x^2 + 15x - 27 = 0$$

$$x^3 - 10x^2 + 15x - 27 = 0$$

$$\lambda = 9 \text{ (by optimization)}$$

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(The above equation is crossed out with a large scribble)

$$\begin{bmatrix} -3 & -3 & 3 \\ 1 & -3 & 1 \\ 6 & -6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -3 & -3 & 3 \\ 1 & -3 & 1 \\ -3 & -3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} -3 & -3 & 3 \\ 1 & -3 & 1 \\ 6 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$6 \begin{bmatrix} 1 & -3 & -3 & 3 & | & 0 \\ 3 & -9 & 3 & 0 & | & 0 \\ 6 & -6 & 0 & 0 & | & 0 \end{bmatrix}$$

$$2x \begin{bmatrix} -3 & -3 & 3 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 6 & -6 & 0 & | & 0 \end{bmatrix}$$

true for

$$-3x_1 - 3x_2 + x_3 = 0$$

$$x_1 = -x_2$$

$$\begin{bmatrix} 0 & -12 & 3 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 6 & -12 & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -12 & 3 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 6 & 0 & -3 & | & 0 \end{bmatrix}$$

$$\lambda = 4$$

$x_1 = x_2$ is an eigen vector if $x_1 = x_2$